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This final report surveys the research accomplishments of a three year effort on the adaptive stabilization and control of distributed parameter systems and on the development of a systematic feedback design methodology for nonlinear control systems. In both areas, significant advances have been made by the use of concepts and techniques from dynamical systems theory, developing enhancements of classical frequency domain ideas (e.g. transmission zeroes, root locus methods, etc.) for both classes of systems. Indeed, for DPS the rigorous development of a root locus theory for parabolic systems is one of our most significant achievements. For nonlinear control design, we have enjoyed three unanticipated breakthroughs. First, the development of a nonlinear enhancement of transmission zeroes enabled the solution of a major open problem in nonlinear control - the nonlinear regulator problem - for systems operating near an equilibrium. Second, the geometric techniques underlying the solution of the regulator problem have been successfully applied to obtain "off-line" feedback laws which solve nonlinear optimal control problems via a nonlinear

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19. analogue of the Riccati equation. Third, these advances, combined with the work described in this report, have led successfully to the development of a nonlinear robust control theory analogous to the development of H^{∞} robust control for linear systems.

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1 Proposed Research Objectives

The main objectives of this three year research effort were the design and analysis of feedback algorithms in adaptive and in nonlinear control. In adaptive control, the principal focuses included adaptive control of distributed parameter systems using fairly simple, low dimensional controllers. An important preliminary to the design of such adaptive algorithms is the design of low dimensional compensators, capable of achieving stabilization or tracking desired trajectories, in the more classical case where the distributed parameter system coefficients are known, modelling for example systems with flexible appendages or components undergoing a slewing maneuver. Research Tasks 2.1-2.8 address specific research tasks which were proposed as part of the long term research effort in adaptive control. Research Tasks 2.1-2.2 proposed the formulation and derivation of (algorithm independent) necessary conditions for the existence of adaptive stabilization and adaptive tracking schemes for distributed parameter systems. Research Task 2.3 focused on the explicit design and analysis of adaptively stabilizing controllers for minimum phase distributed parameter systems with relative degree r. Research Task 2.4 proposes the important extension of Task 2.3 consisting of the analysis and design of adaptive controllers, for the same class of distributed parameter systems, capable of achieving asymptotic disturbance attenuation and of tracking desired reference trajectories. Tasks 2.3-2.4 have focused on analysis and design of adaptive controllers for scalar-input, scalar output distributed parameter systems, while some important applications have access to and require the use of several simultaneous control mechanisms. A challenging problem even in the lumped case, the analysis and design of adaptive controllers for multi-input, multi-output systems, based on the research effort described in Tasks 2.3 -2.4, is proposed in Research Task 2.5. Research Task 2.6 proposed developing systematic methods for modifying the controllers, designed in the research effort required for Tasks 2.3-2.5, to meet specific implementation requirements, such as saturation of feedback gains, guaranteed wind-down, etc.

The sequence of research efforts described in Tasks 2.3-2.6 require the very important preliminary solution of the corresponding design problems for distributed systems whose parameters are unknown. For example, Tasks 2.3-2.4 involve designing adaptive feedback laws capable of stabilization or tracking for minimum phase distributed parameter systems; i.e. for systems whose transmission zeroes lie in the left half complex plane. Research Task 2.7 proposed a research effort leading to the rigorous development of a basic (geometric) theory of transmission zeroes for multi-input, multi-output linear systems, including an intrinsic, coordinate-free characterization of zero dynamics, which have proved so useful

in nonlinear feedback design. The application of the theory of transmission zeroes in the design of feedback laws achieving asymptotic stabilization and output regulation of distributed parameter systems, a fundamental preliminary to adaptive control, was proposed in the final specific research task on adaptive control, Research Task 2.8. Another of the long term research goals of this research effort is the control of nonlinear, distributed parameter systems. For nonlinear systems, control problems such as stabilization about equilibria, stabilization about limit cycles or asymptotic tracking of desired reference trajectories have been largely open problems even for lumped, or finite dimensional, nonlinear systems. Research tasks 3.1-3.5 proposed specific research which focused primarily on proposed efforts directed towards solving such fundamental problems for finite dimensional systems.

Research Task 3.1 proposed a research effort in feedback stabilization of nonlinear control systems. Explicitly, the analysis and design of feedback laws achieving bounded-input, bounded-output stability, a somewhat stronger but highly desirable form of asymptotic stabilization, is proposed. Research Task 3.2 focused on the development of feedback design methods leading to the solution of asymptotic tracking, of stabilization and of linear model matching for nonlinear systems. Research Task 3.3 proposed research directed toward a solution of the nonlinear regulator problem, i.e. the design of a feedback compensator simultaneously achieving asymptotic tracking and disturbance attenuation. There is of course a direct connection between nonlinear systems and adaptive control, since adaptive control schemes themselves are typically nonlinear systems. Past research has shown that stability mechanisms underlying convergence of simple adaptive stabilization schemes is in fact dictated by the stability results for a special class of nonlinear autonomous systems, time varying linear systems. Thus Research Task 3.4 proposed the development of a systematic geometric design methodology for solving problems such as feedback stabilization, disturbance rejection, etc. for linear time- varying systems. Finally, much of the early research on problems such as feedback stabilization or tracking required that certain regularity conditions be satisfied. It is rather typical, however, for special system configurations that these regularity conditions can be violated. Research Task 3.5 addressed this problem, proposing a systematic research effort in the regularization of singular problems arising in nonlinear control.

2 Status of the Research Effort

(References cited are from Section 3)

In our three year effort we have made substantial progress in our research on the control of distributed parameter systems (DPS) and enjoyed several unanticipated major breakthroughs in our research on feedback control of nonlinear systems. The research effort on the control of DPS is a fundamental preliminary to our overall proposed effort in adaptive control; in particular, to the specific research goals delineated in Tasks 2.1-2.8 as described in section 1 and in the proposal. Our work in nonlinear control addresses the specific research goals delineated in the specific Research Tasks 3.1-3.5, which are benchmarks in our longer-term research effort to develop a systematic methodology for the design of feedback laws for nonlinear systems, similar in scope and effectiveness to classical linear system design.

Research Tasks 2.3-2.6 call for the design of explicit, low dimensional adaptive controllers for distributed parameter systems, which should be designed on the basis of fairly intuitive, physical properties of the DPS, particularly frequency domain properties such as system stability (e.g. damping or dissipation) or stability of a system inverse, i.e. the minimum phase property. Research Task 2.1-2.2 thus represent the longer term goal of deriving algorithm independent necessary conditions for the existence of finite-dimensional adaptive controllers for DPS. A very important preliminary to the design of adaptive controllers as envisioned in Research Tasks 2.3-2.6 is a systematic methodology for the design of such feedback strategies, when the system parameters are known. Indeed, Tasks 2.7-2.8, which represent our initial goals in this overall research effort, involve the development of a dynamical systems formulation of the classical concept of systems transmission zeroes and minimum phase properties in terms of which rigorous "geometric" root-locus methods can be developed as a tool for feedback design.

In [8], by developing the notion of "zero dynamics" of a DPS, we sketched an enhancement of root-locus design methods, resolving to a large extent the research problem posed in Task 2.7. In [8], [41], we then applied the concept of "zero dynamics" of a DPS to the design of some explicit feedback laws for stabilization and set-point boundary control for certain problems of heat conduction or wave propagation. Elementary examples [8] also show that a direct generalization of classical root-locus plots fails, even in the case of self-adjoint boundary conditions. This combination of examples showed that while one can be optimistic about the development of a feedback design theory as envisioned in Research Task 2.8, such a theory will be far more subtle than in the classical. lumped parameter

case. Nonetheless, as we sketched for one spatial dimension in [19], with sufficient care one can develop a rigorous, fairly complete enhancement of root-locus design.

The underlying analysis reposes on two new ingredients. The first is based on the discovery that, although the high-frequency gain of a DPS does not exist, its time-domain counterpart – the system instantaneous gain – does of course exist and can be explicitly computed. It is worth noting that the sign of the instantaneous gain is all that is needed to determine the asymptotic behavior of the root-locus plot. In particular, new and even simpler formulas have been found for the instantaneous gain. These new formulas apply to the cases considered in [24], [34] but are also valid for a larger class of control problems including the case of noncolocated actuators and sensors. One special feature of these formulas is that they depend only on the order and coefficients of the highest order terms in the input and output boundary operators rather than the more complicated determinant condition first announced in [34].

The second ingredient, novel to research in DPS, is the application of the classical work by G. D. Birkhoff which enables us to analyze discreteness of the appropriate spectra and completeness of the appropriate eigenfunction expansions, even in the absence of the standard self-adjointness restrictions on the class of boundary controls. These two technical advances, taken together, enabled us to develop a fairly complete root-locus design methodology for parabolic distributed parameter systems, as is described in the forthcoming full-length paper [33]. Furthermore, by using the Dirichlet principle, we have recently found a promising extension of these design techniques to higher spatial dimensions, by appealing to methods from the calculus of variations.

As an example of the scope of the root-locus methods developed for DPS, in [24,33,34] it is shown that, in general, for these DPS all but finitely many of the open loop poles and zeros are real and interlace on the negative real axis. This together with the Hadamard factorization theorem has enabled us to obtain analogs of many other important geometric tools from the finite dimensional root locus theory. For example, the "phase" and "magnitude" criteria from the finite dimensional case also hold in this case . This allows us to obtain such conclusions as a "real axis loci" result from the finite dimensional case: For $k > 0$, points of the root-locus on the real axis lie to the left of an odd number of finite poles and zeroes and for $k < 0$, points of the root-locus on the real axis lie to the left of an even number of finite poles and zeroes.

Also during this period we extended these results to one dimensional hyberbolic problems [50]. For wave and beam problems on a finite interval, we have established boundary feedback control laws using the same tools that were applied in the parabolic case. The problems here are somewhat more interesting since the closed loop poles typically tend to

vertical asymptotes and exponential stability is thus harder to establish. One important tool here has been the use of boundary feedback to render the system equivalent to a dissipative system.

In addition to considering hyperbolic problems on a finite domain we have considered examples on unbounded domains. In this case, quite pathological behavior can be observed even for very simple examples associated with the wave equation. In particular, for the wave equation on a half line [49] with Neumann boundary control, colocated displacement output and a PD control law, we found the unexpected behavior that the point spectrum is empty for all values of the gain parameter except one. At this exceptional value the point spectrum consists of the entire left half plane. Yet the problem remains maximal dissipative with no eigenvalues on the imaginary axis. For this problem, with the absolutely continuous spectrum taken as essential spectrum (cf the classic texts by Kato, or Simon and Reed), this example demonstrates the instability of the essential spectrum subject to compact perturbations. In [49] it is shown that the boundary control effects a rank one perturbation of the spatial operator.

The boundary feedback control laws derived above are in essence output feedback laws, which have the advantage of only requiring on-line processing with just a finite dimensional quantity in memory, as opposed to feedback strategies which need to have access to the entire (infinite-dimensional) state of the DPS. However, even for lumped parameter systems, the problem of determining the extent to which linear system behavior can be influenced by output feedback is open and challenging. Recent techniques and advances in these output feedback problems are the subject of our recent survey [10], an invited paper in honor of J. C. Willems. The references [10] and [25], [30] contain new contributions to this longstanding problem, obtained by using somewhat novel applications of algebraic geometry and Lie theory, respectively. When output measurements do not contain sufficient data to stabilize a DPS, there are several alternatives to obtain enough information about the system state to achieve stabilization. Classical techniques would involve filtering the output to recover the entire (infinite dimensional) state, but the techniques of sampling and multirate sampling can give this kind of information approximately in an on-line manner. References [4], [35]-[40] contain our initial efforts on this inverse problem, giving fairly general results for heat conduction and more general parabolic systems.

Our effort in nonlinear feedback control has concentrated on the research goals outlined in Tasks 3.1-3.5. Task 3.1 is concerned with the design of nonlinear feedback laws which stabilize a given nonlinear control system in the bounded-input, bounded-output (BIBO) sense. The starting point for this research effort effort is our recent design methodology for asymptotic stabilization ([5,9,26]), based on a nonlinear dynamics extension of the concept

of system transmission zeroes - the notion of "zero dynamics." In [20] we used methods from nonsmooth analysis to give a general existence result for zero dynamics in which the control enters in an affine form. From this existence theorem, which seems to be of interest in its own right, we also derived a new necessary condition for feedback stabilization of nonlinear systems [20], stated in terms of an intuitive criterion involving the system zero dynamics. Based on these techniques, we derived in [18] BIBO stabilizing laws for certain broad classes of nonlinear systems.

The underlying methods, consisting of a synthesis of techniques drawn from geometry and nonlinear dynamics, have also been applied to resolve a longstanding problem concerning attitude stabilization of rigid spacecraft, controlled by pairs of gas jets ([23], see [7] simulations). Also, in this direction, we have begun [12] to research the very important problem of designing stabilizing feedback laws so that they are also robust against unmodelled high frequency dynamics. As it turned out the concept of (nonlinear) zero dynamics proved to be as versatile an approach to nonlinear feedback design problems as transmission zeros are for linear automatic control (cf. the survey article [11]). More recently, in [22], we also applied the concept of zero dynamics to the problem of exact linearization of a nonlinear control system by dynamic feedback. Exact linearization of a nonlinear control system by static feedback had been one of the principal methods for designing stabilizing feedback laws for nonlinear systems, although it was well-known that the conditions for exact linearization are very stringent and consequently do not apply to a broad class of nonlinear systems.

Our interest in BIBO stabilization stems from the desire to not only asymptotically stabilize a system, but to maintain Lyapunov stability when the system is required to asymptotically track a bounded reference signal. This problem of non-equilibrium stabilization, i.e. about a bounded but nonconvergent trajectory rather than an equilibrium, lies at the heart of Research Tasks 3.2 and 3.3. Research Task 3.2 focused on the development of feedback design methods leading to the solution of asymptotic tracking, of stabilization and of exact linear model matching for nonlinear systems. As we have cited, our success in Task 3.1 reposed on our successful efforts in developing a fairly general methodology for designing feedback laws which asymptotically stabilize nonlinear systems, as proposed in Task 3.2. An unanticipated solution of both the exact and the asymptotic tracking problems for nonlinear systems, valid near an equilibrium, is given in [3,13,17] and is based on the local solution of the nonlinear regulator problem proposed in Research Task 3.3. Asymptotic tracking in the non-equilibrium case is, however, also of interest , e.g. in the use of tracking a stable limit cycle to enhance BIBO stability in the absence of a stable equilibrium. Preliminary results on this class of problems are given in [15,32], wherein

feedback schemes for tracking limit cycles with sufficiently small amplitude are derived using the methods described above in conjunction with the Hopf Bifurcation Theorem.

Research Task 3.3 is concerned with the important problems of asymptotic disturbance rejection and asymptotic tracking, which when taken together form one of the major tools of control system design, the regulator problem. Thus, the nonlinear regulator problem is the problem of designing a feedback compensator which will ensure that the system to be controlled asymptotically tracks a desired reference signal or trajectory, while at the same time asymptotically rejecting a corrupting disturbance signal which if unattenuated would compromise system performance. Rather unexpectedly, we discovered [13,17,47] necessary and sufficient conditions for local solvability of the nonlinear regulator problem. In [46], these results are illustrated in the feedback design for hover control of a planar model of a vertical take-off and landing aircraft (VTOL). The results on nonlinear regulation contained in reference [17] pertain to systems which are affine in the control variable, which is typical but not universal for applications. In [13] we announce a complete solution of the regulator problems for general nonlinear control systems. Reference [14] shows, by means of analysis and examples, that the solutions to the linear regulator problem is not robust with respect to nonlinear perturbations and, in particular do not solve the output regulation problem for nonlinear systems. The condition is a nonlinear enhancement, using zero dynamics, of an observation in classical automatic control, viz. one can track a sinusoid which oscillates at any frequency which is not a system zero. One important aspect of the results in nonlinear output regulation is explicit construction of the feedback laws derived in the solution of the regulator problem. In particular the nonlinear coefficients or "gains" may be determined "off-line" as the solutions of a nonlinear PDE [26]. This structure is somewhat reminiscent of solutions obtained in the linear quadratic regulator problem, a fact which motivated our recent solution of a class of nonlinear optimal control problems by feedback laws which involve the "off-line" solution of a "Riccati PDE". This is described in [42] for nonlinear systems with quadratic performance measures and in [43] for nonlinear systems with nonlinear but convex performance measures.

The importance of the regulator problem and some of the motivating applications are discussed in Sections 6 and 7. The global solvability of the regulator problem, which is of interest for tracking limit cycles or trajectories over a larger spatial interval, will certainly involve developing more powerful mechanisms for ensuring boundedness of system state trajectories. In our work on altitude stabilization [7,23] and on global asymptotic disturbance rejection [6], a common stability mechanism has emerged, viz., the existence of bounded attractors in the system zero dynamics can be taken advantage of to ensure Lyapunov stability of the closed-loop system. We are currently pursuing this method of

feedback stabilization about attractors [6] as a potential new tool for nonlinear regulation. We have also researched an alternative approach. In [27] we combine the geometric methods described above with the concept of passivity, which has long been a mainstay of stability analysis for nonlinear circuits and which, for the case of lumped linear systems, has a profound frequency domain interpretation easily related to system poles and zeros. Using this combination of techniques, we have developed some apparently powerful, new stability criteria based on a solution to a problem which is a more robust version of feedback equivalence to a linear system, viz. feedback equivalence to a nonlinear passive system. We are currently pursuing this method of feedback equivalence to a passive system as a new tool for nonlinear regulation.

The Ph.D. dissertation completed by S. Pinzoni in the second year of this research effort gives a fairly complete treatment of the research program outlined in Research Task 3.4, which proposed the development of a systematic geometric design methodology for solving problems such as feedback stabilization, disturbance rejection, etc. for linear time-varying systems. Indeed, in his thesis Pinzoni develops the geometric notions of controlled invariant subspace, zero-dynamics, relative degree and instantaneous gain, applying them to the solution of the fundamental problems of disturbance decoupling (DDP), stabilization by static and dynamic output feedback and disturbance decoupling with stability (DDPS).

Research Task 3.5 addresses the fact that it is rather typical that the regularity conditions assumed in much of the early research on problems - such as feedback stabilization or tracking - can be violated for special system configurations. References [20], [21] and [28] document the initial phase of our systematic research effort in the regularization of singular problems arising in nonlinear control. In [20], using methods from nonsmooth analysis, we are able to define and prove the existence of zero dynamics for nonlinear systems in which the control enters in an affine form, a result which has already shown to be of some importance in nonlinear feedback design. In [21] we apply techniques from singular perturbation theory to analyze, in the scalar input-scalar output case, the singular behavior of zero dynamics when the system relative degree changes as a function of a parameter. Reference [28] treats this problem in detail in the multivariable case.

In the course of our research program we have also researched some of the more basic aspects of nonlinear systems and control, which we feel are or will soon be important to fundamental problems of estimation and control. In [1] we investigated a novel use of harmonic analysis to study the problem of observability, i.e. recoverability of the system state from system observations, for a certain class of ergodic systems. State recovery in a noisy environment has been solved for linear systems by the method of Kalman Filtering, provided of course the observations are truly generated by an underlying linear system of

the correct dimension. When the system which generates the observation data is nonlinear or of higher dimension (e.g. a DPS), the asymptotic behavior of the Kalman Filter is not understood. In references [2], [16] and [29], [31] we apply methods from nonlinear dynamics to obtain some rather precise information about the steady-state behavior of the Kalman Filter, as a nonlinear dynamical system, in these cases. Reference [16] addresses the problem of parameterizing those shaping filters, of a fixed dimension, which could produce observed signals having an a priori set of statistics, e.g. a fixed number of given correlation coefficients.

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[41] D.S. Gilliam and C.I. Byrnes, "Stability of certain distributed parameter systems by low dimensional controllers: a root locus approach," *Proceedings 29th IEEE International Conference on Decision and Control.*

[42] C.I. Byrnes, "New methods for nonlinear optimal control," *Proceedings of First European Control Conf.*, Grenoble 1991.

[43] C.I. Byrnes, "Some partial differential equations arising in nonlinear control and optimization," Birkhäuser-Boston, (1991).

[44] F. Delli Priscoli, A. Isidori, "Robust tracking for a class of nonlinear systems," *Proc. 1st European Control Conf.* Grenoble, (1991).

[45] A. Isidori, "Feedback control of nonlinear systems," *Proc. 1st European Control Conf.*, Grenoble, (1991).

[46] C.I. Byrnes and J. Roltgen, "Hover Control of a PVTOL Using Nonlinear Regulator Theory," *Proc. of 1991 ACC*, Boston.

[47] C.I. Byrnes and A. Isidori, "Asymptotic Tracking and Disturbance Rejection in Nonlinear Systems *New Trends in Systems Theory*, Birkhauser-Boston, 1991.

[48] C.I. Byrnes, "The solution of nonlinear Lagrange and Bolza problems via Riccati partial differential equations," in preparation.

[49] C.I. Byrnes and D.S. Gilliam, "An interesting example in boundary control for a hyperbolic problem on an unbounded domain," (preprint).

[50] C.I. Byrnes and D.S. Gilliam, "Boundary feedback control for hyperbolic systems," in preparation.

4 Participating Professionals

Principal Investigator:

Dr. Christopher I. Byrnes

Post-doctoral Researcher

Dr. David Gilliam

Ph.D. Students

X.M. Hu, Ph.D. May 1989, Arizona State University,

Thesis Title: Robust Stabilization of nonlinear systems.

S. Pinzoni, Ph.D.,December 1989, Arizona State University

Thesis Title: Stabilization and control of linear time-varying systems.

M. Lei, Ph.D. candidate, Washington University.

S. Pandian, Ph.D. candidate, Washington University.

5 Scientific Interactions

September 1988

Uniformly bounded-input, bounded output stability, Department of Mathematics, Texas Tech University.

October 1988

Geometric methods for feedback design for nonlinear control system, (Plenary lecture), Midwest Symposium on Differential Equations, Iowa State University, Ames, Iowa

November 1988

Feedback stabilization about attractors, Department of Systems and Decision Sciences, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Feedback stabilization about attractors, Department of Electrical Engineering, University of Padova, Padova, Italy.

December 1988

Feedback stabilization about attractors and asymptotic disturbance rejection for nonlinear systems. Invited lecture, 28th IEEE Conference on Decision and Control, Austin Texas.

Attitude stabilization of rigid spacecraft: stability and instability near attractors, Invited lecture, 28th IEEE Conference on Decision and Control, Austin, Texas.

Asymptotic properties of root-loci for distributed parameter systems, Invited lecture presented by Dr. David Gilliam at 28th IEEE Conference on Decision and Control.

January 1989

Feedback stabilization about attractors, Department of Mathematics, University of Paris-Dauphine, Paris, France.

Feedback stabilization about attractors, Laboratory for Signals and Systems, CNRS, Gif-sur-Yvetter, France.

March 1989

Geometric methods for nonlinear feedback systems, ESA Lecture Series (three lectures) Texas Tech University, Lubbock, Texas.

April 1989

Feedback stabilization about attractors, Nonliner Sciences Institute, University of California-Davis, Davis, California.

Output regulation for nonlinear systems, Department of Mathematics, Montana State University, Bozeman, Montana.

May 1989

Asymptotic stabilization of nonlinear minimum phase systems, SIAM conference on Control in the 90's, San Francisco, California.

Geometric methods for nonlinear feedback design, a short course (5 lectures) Department of Optimization and Systems Theory, Royal Institute of Technology, Stockholm, Sweden.

June 1989

Steady-state response and asymtotic tracking for nonlinear systems, IIASA Conference on Nonlinear Synthesis, Sopron, Hungary.

Output regulation for nonlinear systems, Plenary lecture, IFAC Symposium on Nonlinear Control Systems, Capri, Italy.

Asymptotic tracking and disturbance rejection for non-affine nonlinear control systems. Invited lecture, 1989 International Symposium on Mathematical Theory of Networks and Systems, Amsterdam, The Netherlands.

The cohomology of moduli spaces of controllable linear systems, Invited lecture 1989 International Symposium on Mathematical Theory of Networks and Systems, Amsterdam, The Netherlands.

Output regulation of nonlinear systems, 1989 International Symposium on Mathematical Theory of Networks and Systems, Amsterdam, The Netherlands. Invited lecture presented by Dr. A. Isidori.

Boundary feedback stabilization of distributed parameter systems, Invited lecture presented by Dr. David Gilliam at 1989 International Symposium on Mathematical Theory of Networks and Systems, Amsterdam, The Netherlands.

September 1989

Geometric Methods for the Design of Nonlinear Feedback Systems, International Conference on Optimization and Optimal Control, Baikal, USSR. Plenary Lecture.

Feedback Stabilization of Nonlinear Systems, International Conference on Optimization and Optimal Control, Baikal, USSR. Plenary Lecture.

Geometric Methods for the Design of Nonlinear Feedback Systems, Institute for Problems of Control, Moscow, USSR. Invited Colloquium.

Geometric Methods for the Design of Nonlinear Feedback Systems, Steklov Institute of Mathematics, Moscow, USSR. Invited Colloquium.

Geometric Methods for the Design of Nonlinear Feedback Systems, Glushkov Institute for Cybernetics, Kiev, USSR. Invited Colloquium.

November 1989

Discrete Observability for Parabolic Systems, Society of Industrial and Applied Mathematics, Conference on Control in the 90's, San Francisco, 1989. Invited paper, presented by Dr. D.S. Gilliam

Output Regulation of Nonlinear Systems, Department of Electrical and Computer Engineering, Univ. of Texas -Austin. Invited Colloquium.

December 1989

Asymptotic Tracking and Disturbance Attenuation for Nonlinear Systems, 7th Southwest Symposium on Systems and Control, Lubbock, Texas. Invited Paper.

Root Locus for Distributed Parameter Systems, 7th Southwest Symposium on Systems and Control, Lubbock, Texas. Invited Paper, presented by Dr. D.S.Gilliam.

Steady-state Response and Asymptotic Tracking for Nonlinear Systems, 28th IEEE Conf. on Dec. and Contr., Ft. Lauderdale. Invited Paper.

January 1990

Steady-state Response and Asymptotic Tracking for Nonlinear Systems, Dept. of Optimization and System Theory, Royal Institute of Technology, Sweden. Invited Colloquium.

February 1990

Output Regulation of Nonlinear Systems, Center for Systems Science and Engineering Research, Arizona State University. Invited Colloquium.

March 1990

Technical Briefing on Nonlinear Aspects of Flight Control Systems and Computational Fluid Dynamics, McDonnell-Douglas Aircraft Co., St. Louis.

Output Regulation of Nonlinear Systems, LADSEB-CNR, Padova, Italy. Invited Colloquium.

Root Locus Methods for Boundary Feedback Stabilization of Parabolic Distributed Parameter Systems, Università di Padova, Dipartimento di Elettronica e Informatica. Invited Colloquium presented by D.S. Gilliam.

Root Locus Methods for Boundary Feedback Stabilization of Parabolic Distributed Parameter Systems, Università di Roma, "La Sapienza", Dipartimento di Informatica e Sistemistica. Invited Colloquium presented by D.S. Gilliam.

April 1990

An Invitation to Nonlinear Control, Dept. of Electrical and Computer Engineering, Univ. of Illinois. Invited Address in the Graduate Colloquium Series.

Output Regulation of Nonlinear Systems, Coordinated Sciences Laboratory, Univ. of Illinois. Invited Colloquium.

May 1990

A Riccati Partial Differential Equation for Nonlinear Optimal Control, Dept. of Optimization and System Theory, Royal Institute of Technology, Sweden. Invited Colloquium.

Output Regulation of Nonlinear Systems, Dept. of Electrical and Computer Engineering, Univ. of Linköping, Sweden. Invited Colloquium.

June 1990

Controlled Invariance, Zero Dynamics, and Viability Domains for Nonlinear Control Systems, 7th INRIA Conf. on Analysis and Optimization of Systems, Anibes, France. Invited Paper.

July 1990

An Introduction to Nonlinear Control, Summer Institute of the Chalmers University, Stockholm, Sweden. Invited Short Course.

Passive, Positive Real and Minimum Phase Nonlinear Systems, Conference on Nonlinear Control Systems, Lyons, France. Plenary Lecture.

Necessary Conditions for Feedback Stabilization of Nonlinear Control Systems, Conference on New Trends in System Theory, Genova, Italy. Invited Paper.

August 1990

Partial Differential Equations Arising in Nonlinear Control, 2nd Conference on Computation and Control, Bozeman, Montana. Plenary Lecture.

Root Locus Methods for Boundary Feedback Stabilization of a Parabolic Distributed Parameter Systems, Conference on Computation and Control, Bozeman, Montana. Invited lecture presented by Dr. D.S. Gilliam.

September 1990

Stable, Unstable and Center Manifolds for Fast Filtering Algorithm, IIASA Conference on the Modeling and Control of Uncertain Systems, Soporn, Hungary. Plenary Lecture.

October 1990

Partial Differential Equations Arising in Nonlinear Control, Mathematics Institute, Universiteit Groningen, The Netherlands, Invited Colloquium.

November 1990

Output Regulation of Nonlinear Systems, Honweywell, Minneapolis, Invited Lecture.

Passive, Positive Real and Minimum Phase Nonlinear Systems, University of Minnesota, Invited Colloquium.

December 1990

Stabilization and Output Regulation of Nonlinear Systems in the Large, 29th IEEE Conference on Decision and Control, Honolulu, Invited paper.

Stabilization of Certain Distributed Parameter Systems by Low Dimensional Controllers: A Root Locus Approach, 29th IEEE Conference n Decision and Control, Invited Paper delivered by Dr. Gilliam.

Exact Linearization of Zero Dynamics, 29th IEEE Conference on Decision and Control, Honolulu, Invited Paper delivered by Dr. Isidori.

February 1991

Partial Differential Equations Arising in Nonlinear Control, Center for Systems Science and Engineering Research, Arizona State University. Invited Colloquium.

March 1991

Partial Differential Equations Arising in Nonlinear Control, LADSEB-CNR, Padova, Italy. Invited Colloquium.

April 1991

Recent Advances in Nonlinear Control, AFOSR Workshop on Turbulence and Flow Control, Ohio State University, Plenary Lecture.

Partial Differential Equations Arising in Nonlinear Control, Department of Electrical Engineering, Linköping University, Sweden. Invited Colloquium.

May 1991

Partial Differential Equations Arising in Nonlinear Control, Ecole-Normale Superieure, Paris. Invited Colloquium.

Partial Differential Equations Arising in Nonlinear Control, CNS Laboratory, Fontainbleu. Invited Colloquium.

June 1991

Hover Control of a PVTOL Aircraft, ACC, Boston. Invited paper given by coauthor, Mr. John Roltgen (MDC).

Partial Differential Equations Arising in Nonlinear Control, Laboratiore des Signaux et Systemes, CNRS, Gif-sur-Vette. Invited Colloquium.

Partial Differential Equations Arising in Nonlinear Control, INRIA-Antibes. Invited Colloquium.

Robust Output Regulation of Nonlinear Systems, MTNS-91, Kobe, Japan. Plenary lecture delivered by Dr. Isidori

July 1991

New Methods for Nonlinear Optimal Control, 1st European Control Conference, Grenoble. Invited Paper.

Passive, Positive Real and Minimum Phase Nonlinear Systems, Department of Optimization and System Theory, Royal Institute for Technology, Stockholm. Invited Colloquium.

Robust Output Regulation of Nonlinear Systems, 1st European Control Conference, Grenoble. Plenary lecture delivered by Dr. Isidori

6 New Discoveries

During the first year of this research program, significant progress was made principally on two research fronts, boundary feedback control of distributed parameter systems and feedback control of nonlinear systems. Motivating applications from our work on the control of distributed parameter systems include the stabilization of a rigid body with flexible appendages undergoing a slewing maneuver. In this area, the research developed so far has uncovered the theoretical underpinnings of root-locus plots and root-locus design methods, which are graphical stability and design criteria used quite freely in aerospace engineering. It has often been noted that such root-locus methods do not always work for distributed parameter systems as they do for classical lumped parameter systems. For this reason, there is a real need for a rigorous understanding of and a clear delineation of the scope of these potentially very powerful, intuitive graphical tools for analysis and design of distributed parameter systems. The underlying analysis reposes on two new ingredients. The first is based on the discovery that, although the high-frequency gain of a DPS does not exist, its time-domain counterpart - the system instantaneous gain - does of course exist and can be explicitly computed. It is worth noting that the sign of the instantaneous gain is all that is needed to determine the asymptotic behavior of the root-locus plot. To this end, fairly simple formulas have been found for the instantaneous gain which apply to the cases considered in [24], [34] but are also valid for a larger class of control problems including the case of noncolocated actuators and sensors. One special feature of these formulas is that they depend only on the order and coefficients of the highest order terms in the input and output boundary operators rather than the more complicated determinant condition first announced in [34].

The second ingredient, novel to research in DPS, is the application of the classical work at the beginning of this century by G. D. Birkhoff which enables us to analyze discreteness of the appropriate spectra and completeness of the appropriate eigenfunction expansions, even in the absence of the standard self-adjointness restrictions on the class of boundary controls. These two technical advances, taken together, enabled us to develop a fairly complete root-locus design methodology for parabolic distributed parameter systems, as is described in the forthcoming full-length paper [33].

In the second area, we have enjoyed an unanticipated solution of one of the major problems in nonlinear control, a solution of the nonlinear regulation problem. This is the problem of designing a feedback compensator which will enable the system to be controlled to asymptotically track a desired reference signal or trajectory, while at the same time asymptotically rejecting an unwanted disturbance signal which if unattenuated

would severely compromise system performance. Mathematically, simple system theoretic necessary and sufficient conditions for the solution of the nonlinear regulator problem have been determined, valid for local deviations from an equilibrium.

From a feedback design point of view, beyond the rigorous solution of the nonlinear regulator problem, a very important component of this advance is the actual form of the feedback compensation, which appears to be computationally tractable. In close analogy with the linear case, the structure of the feedback law is that a nonlinear proportional error compensator where the nonlinear feedback "gains" can be computed "off-line" by solving a nonlinear partial differential equation, quite similar to very important roles played by the Riccati equation in linear systems design. This structure is somewhat reminiscent of solutions obtained in the linear quadratic regulator problem, a fact which motivated our second advance in nonlinear feedback design – our recent solution of a class of nonlinear optimal control problems by feedback laws which involve the "off-line" solution of a "Riccati PDE". This is described in [42] for nonlinear systems with quadratic performance measures and in [43] for nonlinear systems with nonlinear but convex performance measures.

The basic research on the regulator problem and on the solvability of steady-state Riccati PDE's also provided the starting point for our third significant breakthrough in nonlinear control system design – the development of nonlinear H^∞ robust control techniques.

7 Additional Information

As described in Section 1 and the specific research tasks described in our proposal, our work on distributed parameter control is a preliminary to one of our longer range research goals, the adaptive control of linear and nonlinear distributed parameter systems. Even for lumped systems, serious problems remain to be researched before adaptive control can be used with confidence for high performance DOD systems, despite the remarkable commercial success of adaptive controllers for slower systems such as those arising in process control applications. One of the motivating applications for our basic research in adaptive and nonlinear control is flow control, e.g. the control of fluid flow across an airfoil using thermal actuators, and related topics such as combustion and noise control currently being researched in experimental laboratories in the United States and Japan. Here , more complicated dynamical behavior is exhibited in both the process to be controlled and the actuator. While there is a definite need, for the purpose of control, for the development of specific alternative models to the Navier Stokes equations, it is extremely likely that simplified models exhibiting some of the complicated behavior encountered in turbulent fluid flow will involve nonlinear and also infinite dimensional systems. Indeed, the recent "Fleming Report" on Future Directions in Control Theory emphasized research on flow control as an important research need for the future of American technology.

The problems of stabilizing and controlling nonlinear systems are limiting factors in the design of several DOD systems. For example, there is current research and development effort in the aerospace industry dedicated toward stabilization and control of high performance aircraft operating in nonlinear flight conditions involving agility and high angles-of-attack. Because linear systems exhibit much more predictable and well-understood behavior, the control of linear systems has been more highly developed than the control of nonlinear systems. For this reason, current approaches to flight control in the presence of nonlinear effects, e.g. "gain scheduling", have typically involved finding an "equivalent" linear system, for which a controller is then designed using existing linear methods. However, for more highly nonlinear maneuvers involving increased agility and higher angle-of-attack, the limitations of conventional design methods stem from the lack of a reasonable "equivalent" linear system which incorporates in some way the increasingly dominant nonlinear effects. This research effort in nonlinear stabilization and control is aimed at developing a systematic methodology to overcome some of these limitations. The Fleming Report also emphasized the importance of basic problems such as nonlinear feedback stabilization to the future of the American research effort in control, underscoring the earlier consensus of the 1986 IEEE Santa Clara meeting which stated that "nonlinear

feedback stabilization is by far... the most important open problem in nonlinear control."

The recent success of this research effort in the local solution of the nonlinear regulator problem relies heavily on our earlier AFOSR sponsored research on the problem of nonlinear feedback stabilization. In fact, in 1989, the principal investigator was honored as a Fellow of the IEEE for his "contributions to the feedback stabilization and the control of linear and nonlinear systems," an honor limited to a small fraction of the IEEE membership. The research on the nonlinear regulator problem, and an application to hover control for a simplified model of a vertical take-off and landing aircraft, were highlighted in the 1991 publication of the AFOSR Research Accomplishments reviews. Since that time, the foundational paper on output regulation, which was published in the IEEE Transactions on Automatic Control was nominated for the George Axelby Award for the best paper published in the Transactions in the years 1989-1990, an award the paper subsequently won in December 1991 at the IEEE Conference on Decision and Control in Brighton, England.